DAY TWELVE

Waves

Learning & Revision for the Day

- Wave Motion
- Speed of Waves
- Sound Waves
- Displacement Relation for a Progressive or Harmonic Wave
- Principle of Superposition of Waves
- Reflection and Transmission of Waves
- Standing or Stationary Waves
- Beats
- Doppler's Effect

Wave Motion

Wave motion involves transfer of disturbance (energy) from one point to the other with particles of medium oscillating about their mean positions i.e. the particles of the medium do not travel themselves along with the wave. Instead, they oscillate back and forth about the same equilibrium position as the wave passes by. Only the disturbance is propagated.

1. **Longitudinal Waves** When particles of the medium vibrate parallel to the direction of propagation of wave, then wave is called **longitudinal wave**. These waves propagate in the form of compressions and rarefactions. They involve changes in pressure and volume. The medium of propagation must possess elasticity of volume. They are set up in solids, liquids and gases.
2. **Transverse Waves** When the particles of the medium vibrate in a direction perpendicular to the direction of propagation of wave, then wave is called transverse waves. These wave propagtes in the form of crests and troughs. These waves can be set up in solids, on surface of liquids but never in gases.

Terms Used in Wave Motion

- **Angular Wave Number** Number of wavelength in the distance 2π is called the wave number or propagation constant.

$$K = \frac{2\pi}{\lambda} \text{ rad/m}$$

- **Particle velocity** It is the velocity of the particle executing simple harmonic motion.

i.e. $$v = \frac{dy}{dt}$$

where, y denotes displacement at any instant.

- **Wave Velocity** The velocity of transverse wave motion is given by

$$v = \frac{\text{Distance travelled by wave}}{\text{Time taken}}$$

i.e. $$v = \frac{\lambda}{T} = \left(\frac{1}{T}\right)\lambda = \frac{\omega}{x} \quad \text{or} \quad v = \nu\lambda$$

- **Differential Equation of Wave Motion**

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$

Speed of Waves

Speed of waves is divided in two types as per the nature of wave, these are given below

1. Speed of Transverse Wave

The expression for speed of transverse waves in a solid and in case of a stretched string can be obtained theoretically

- In solids, $v = \sqrt{\frac{\eta}{d}}$

  where, η is the modulus of rigidity and d is the density of the medium.

- In a stretched string, $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{Mg}{\pi r^2 d}}$

  where, T = the tension in the string,
  m = the mass per unit length of the string,
  M = mass suspended from the string,
  r = radius of the string and
  d = density of the material of the string.

2. Speed of Longitudinal Wave (or Sound Wave)

Following are the expressions for the speed of longitudinal waves in the different types of media

- If the medium is solid,

$$v = \sqrt{\frac{B + \frac{4}{3}\eta}{\rho}}$$

  where B, η and ρ are values of bulk modulus, modulus of rigidity and density of the solid respectively.
  If the solid is in the form of a long rod, then

$$v = \sqrt{\frac{Y}{\rho}}$$

  where, Y is the Young's modulus of the solid material.

- In a liquid,

$$v = \sqrt{\frac{B}{\rho}}$$

  where B is the bulk modulus of the liquid.

- According to Newton's formula, speed of sound in a gas is obtained by replacing B with initial pressure p of the gas i.e. $B = p$.

$$v = \sqrt{\frac{p}{\rho}}$$

Factors Affecting Speed of Sound

- **Effect of Temperature on Velocity** With rise in temperature, the velocity of sound increases as

$$v = \sqrt{\frac{\gamma RT}{M}}; \quad \text{i.e. } v \propto \sqrt{T}; \quad \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

  Speed of sound in air increases by 0.61 m/s for every 1°C rise in temperature.

- **Effect of Pressure for Gase Medium** Pressure has no effect on the velocity of sound, provided temperature remains constant.
- **Effect of Humidity** When humidity in air increases, its density decreases and so velocity of sound increases.

  For solids, $v = \sqrt{\frac{Y}{D}}$. For liquids, $v = \sqrt{\frac{B}{D}}$

  where, Y = Young's modulus of elasticity
  B = bulk modulus of elasticity.

Sound Waves

The longitudinal waves which can be heard are called sound waves.

They are classified into following categories

- **Infrasonics** The longitudinal waves having frequencies below 20 Hz are called infrasonics. These waves cannot be heard. These waves can be heard by snakes.
- **Audible waves** The longitudinal waves having the frequency between 20 Hz and 20000 Hz are called audible waves. Human can hear these waves.
- **Ultrasonics** The longitudinal waves having the frequencies above 20000 Hz are called ultrasonics. These waves are also called supersonic waves or supersonics.

Displacement Relation for a Progressive or Harmonic Wave

The equation of a plane progressive or simple harmonic wave travelling along positive direction of x-axis is

$$y = a\sin(\omega t - kx) \Rightarrow y = a\sin\frac{2\pi}{T}\left(t - x\frac{T}{\lambda}\right).$$

$$\Rightarrow \quad y = a\sin\frac{2\pi}{\lambda}(vt - x) \Rightarrow y = a\sin\omega\left(t - \frac{x}{v}\right)$$

$$\Rightarrow \quad y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right).$$

- If maximum value of $y = a$, i.e. a is amplitude, then

  dy/dt = velocity of particle

$$v = \frac{dy}{dt} = a\omega\cos\cdot\frac{2\pi}{\lambda}(vt - x)$$

$$\left(\frac{dy}{dt}\right)_{max} = \frac{2\pi va}{\lambda} = 2\pi na = \omega a \qquad \text{[where, } n = \text{frequency]}$$

- Acceleration of particle

$$\frac{d^2y}{dt^2} = -\omega^2 a \sin\frac{2\pi}{\lambda}(vt - x)$$

- For a wave, velocity of sound wave

$$v = \text{frequency } (n) \times \text{wavelength } (\lambda)$$

$\Rightarrow$ $$v = n\lambda$$

- Angular speed, $\omega = 2\pi n = \frac{2\pi}{T} \Rightarrow \omega = \frac{2\pi v}{\lambda}$

Relation between Phase Difference, Path Difference and Time Difference

- Phase difference $(\phi) = \frac{2\pi}{\lambda} \times$ path difference (x)

$\Rightarrow$ $$\phi = \frac{2\pi x}{\lambda} \Rightarrow x = \frac{\phi\lambda}{2\pi}$$

- Phase difference $(\phi) = \frac{2\pi}{T} \times$ time difference (t)

$\Rightarrow$ $$\phi = \frac{2\pi t}{T} \quad \Rightarrow t = \frac{T\phi}{2\pi}$$

- Time difference $(t) = \frac{T}{\lambda} \times$ path difference (x)

$\Rightarrow$ $$t = \frac{Tx}{\lambda} \Rightarrow x = \frac{\lambda t}{T}$$

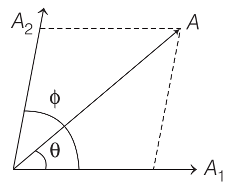
Principle of Superposition of Waves

Two or more waves can traverse the same space independently of one another. The resultant displacement of each particle of the medium at any instant is equal to the vector sum of displacements produced by the two waves separately. This principle is called principle of superposition of waves.

$$y = y_1 + y_2 + y_3 + \ldots..$$

Interference of Waves

When two waves of same frequency (or same wavelength) travelling along same path superimpose each other, there occurs redistribution of energy in the medium. At a given position (x being constant) displacement due to two waves be



$$y_1 = A_1 \sin\omega t \quad \text{and} \quad y_2 = A_2 \sin(\omega t + \phi)$$

Then, resultant displacement

$$y = y_1 + y_2 = A\sin(\omega t + \phi)$$

where, $$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

and $$\tan\theta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$$

Constructive and Destructive Interference

- When the wave meet a point with some phase, **constructuve interference** is obtained at that point.
  - (i) Phase difference between the waves at the point of observation $\phi = 0°$ or $2\pi n$.
  - (ii) Resultant amplitude at the point of observation will be maximum, $A_{max} = A_1 + A_2$.
- When the waves meet a point with opposite phase, **destructive interference** is obtained at that point.
  - (i) Phase difference between the waves at the point of observation $\phi = 180°$ or $(2n-1)\pi$.
  - (ii) Resultant amplitude at the point of observation will be minimum, $A_{min} = A_1 - A_2$.

Intensity

The intensity of waves is the average amount of energy transported by the wave per unit area per second normally across a surface at the given point.

Intensity $(I_1) \propto$ (Amplitude $A)^2$

$\therefore$ $$\frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2$$

If I_1 and I_2 are intensities of the interfering waves and ϕ is the phase difference, then resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2} = (\sqrt{I_1} + \sqrt{I_2})^2, \text{ for } \phi = 2\pi n$$

and $$I_{min} = I_1 + I_2 - 2\sqrt{I_1I_2}$$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2, \text{ for } \phi = (2n+1)\pi$$

Power

If P is power of a sound source, then intensity (I) follows inverse square law of distance (d).

$$I = \frac{P}{4\pi d^2}$$

Reflection and Transmission of Waves

When sound waves are incident on a boundary separating two media, a part of it is reflected back into the initial medium while the remaining is partly absorbed and partly transmitted into the second medium.

Standing or Stationary Waves

Standing or stationary wave is formed due to superposition of two progressive waves of same nature, same frequency (or same wavelength), same amplitude travelling with same speed in a bounded medium in mutually opposite directions.

If the incident wave be represented as $y_1 = A\sin(\omega t - kx)$

and the reflected wave as $y_2 = A\sin(\omega t + kx)$,

then $y = y_1 + y_2 = A\sin(\omega t - kx) + A\sin(\omega t + kx)$

$\Rightarrow$ $y = 2A\cos kx \sin\omega t$

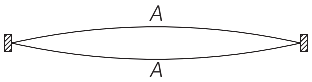
The resultant wave does not represent a progressive wave.

Standing Waves in String

Consider a string of length L stretched under tension T between two fixed points (i.e. clamped at its ends). Transverse wave is set up on the string whose speed is given by $v = \sqrt{T/\mu}$, where μ is the mass per unit length of the string.

Different modes of vibration of stretched string are discussed below

- Let only one anti-node A is formed at the centre and string vibrates in one segment only, it is called **fundamental mode**, then



Fundamental or first harmonic

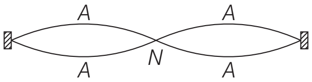
$$L = \frac{\lambda_1}{2} \text{ or } \lambda_1 = 2L$$

Frequency of vibration in fundamental mode

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

It is known as the **fundamental frequency** or **first harmonic**.

- If string vibrates in two segments, then



Second harmonic

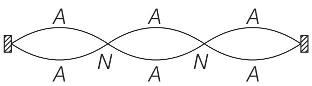
$$L = \lambda_2$$

and $\nu_2 = \frac{v}{\lambda_2} = \frac{1}{L}\sqrt{\frac{T}{\mu}} = 2\nu_1$

It is known as **first overtone** or **second harmonic.**

- If the string vibrates in three segments,

then $L = \frac{3\lambda_3}{2}$



Third harmonic

and $\nu_3 = \frac{v}{\lambda_3} = 3\nu_1$

It is called **second overtone** or **third harmonic**.

- In general, if a string vibrates in p segments [i.e. have $(p + 1)$ nodes and p antinodes],

then $\nu_{pth} = \frac{p}{2L}\sqrt{\frac{T}{\mu}} = p\nu_1$

and it is known as pth harmonic or $(p - 1)$th overtone.

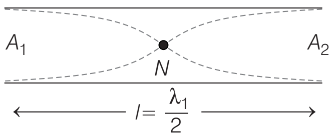
Standing Waves in Organ Pipes (Air Columns)

Organ pipes are those cylindrical pipes which are used for producing musical (longitudinal) sounds. The standing waves in both organ pipes (i.e. open organ pipe and closed organ pipe) are described below.

1. Open Organ Pipe

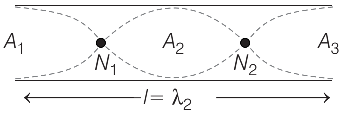
In an open organ pipe, always anti-node is formed at both open ends. Various modes of vibration of air column in an open organ pipe are shown below

- First harmonic $l = \frac{\lambda_1}{2} \Rightarrow f_1 = \frac{v}{2l}$



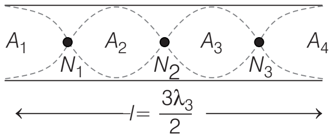
(a)

- Second harmonic or first overtone $l = \lambda_2$; $f = \frac{2v}{2l}$



(b)

- Third harmonic or second overtone $l = \frac{3\lambda_3}{2}$; $f = \frac{3v}{2l}$



(c)

- All harmonics are present in open pipe with their frequencies in the ratio $1:2:3:4 \ldots$ and ratio of overtones $= 2 : 3 : 4 : 5 \ldots$

  Position of nodes from one end $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \ldots$

  Position of anti-nodes from one end

  $$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \ldots$$

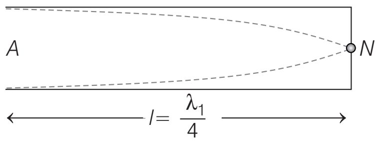
2. Closed Organ Pipe

In a chosed organ pipe, always node is formed at the closed end. Various mode of vibration of air column in a closed organ pipe are shown below

- First harmonic

  $l = \frac{\lambda_1}{4}$

  $f = \frac{v}{4l}$

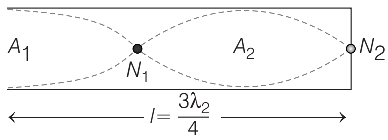


(a)

- Third harmonic (first overtone)

  $l = \frac{3\lambda_3}{4}$

  $f_3 = \frac{3v}{4l}$

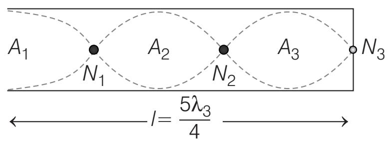


(b)

- Fifth harmonic (second overtone)

  $l = \frac{5\lambda_5}{4}$;

  $f_5 = \frac{5v}{4l}$



(c)

- In closed organ pipe only odd harmonics are present. Ratio of harmonic is $n_1 : n_3 : n_5 = 1:3:5$.

- Ratio of overtones $= 3:5:7$
- Position of nodes from closed end $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$
- Position of antinodes from closed end $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$

Beats

When two sound waves of nearly equal (but never equal) or slightly different frequencies and equal or nearly equal amplitudes travelling along the same direction superimpose at a given point, the resultant sound intensity alternately rises and falls. This alternate rise and fall of sound at a given position is called **beats**.

- Number of beats formed per second is called the frequency of beats. If two sound waves of frequencies ν_1 and ν_2 superimpose, then frequency of beats $= (\nu_1 \sim \nu_2)$, i.e. either $(\nu_1 - \nu_2)$ or $(\nu_2 - \nu_1)$.
- For formation of distinct beats, the difference between the frequencies of two superimposing notes should be less than 10 Hz.
- Our perception of loudness is better co-related with the second level measured in decibel (dB) and defined as follows

  $\beta = 10 \log_{10}\left(\frac{I}{I_0}\right)$, where $I_0 = 10^{-12}$ Wm^2 at 1kHz.

Tuning Fork

The tuning fork is a metallic device that produces sound of a single frequency.

Suppose, a tuning fork of known frequency n_A is sounded together with another tuning fork of unknown frequency (n_B) and x beats heard per second.

There are two possibilities to know frequency of unknown tuning fork

$$n_A - n_B = x \quad \ldots(\text{i})$$
$$n_B - n_A = x \quad \ldots(\text{ii})$$

We can find true frequency of tuning fork B from a pair of tuning forks A and B, in which frequency of A is known and where x is the beats per second.

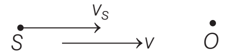
When B is loaded (its frequency decreases)		**When B is filled** (its frequency increases)	
(i)	If x increases, then $n_B = n_A - x$	(i)	If x increases, then $n_B = n_A + x$
(ii)	If x decreases, then $n_B = n_A + x$	(ii)	If x decreases, then $n_B = n_A - x$
(iii)	If x remains same, then $n_B = n_A + x$	(iii)	If x remains same, then $n_B = n_A - x$
(iv)	If x becomes zero, then $n_B = n_A + x$	(iv)	If x becomes zero, then $n_B = n_A - x$

Doppler's Effect

The phenomena of apparent change in frequency of source due to a relative motion between the source and observer is called Doppler's effect.

- **When Source is Moving and Observer is at Rest** When source is moving with velocity v_s, towards an observer at rest, then apparent frequency

  $$n' = n\left(\frac{v}{v - v_s}\right)$$

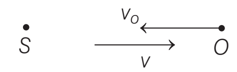


  If source is moving away from observer, then

  $$n' = n\left(\frac{v}{v + v_s}\right)$$

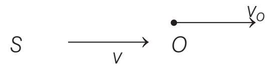
- **When Source is at Rest and Observer is Moving** When observer is moving with velocity v_o, towards a source at rest, then apparent frequency.

  $$n' = n\left(\frac{v + v_o}{v}\right)$$



  When observer is moving away from source, then

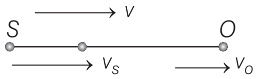
  $$n' = n\left(\frac{v - v_o}{v}\right)$$



- **When Source and Observer Both are Moving**

  (a) When both are moving in same direction along the direction of propagation of sound, then

  $$n' = n\left(\frac{v - v_o}{v - v_s}\right)$$

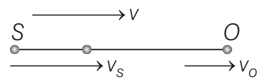


  (b) When both are moving in same direction opposite to the direction of propagation of sound, then

  $$n' = n\left(\frac{v + v_o}{v + v_s}\right)$$

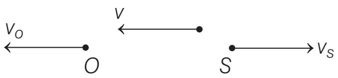
  (c) When both are moving towards each other, then

  $$n' = n\left(\frac{v + v_o}{v - v_s}\right)$$



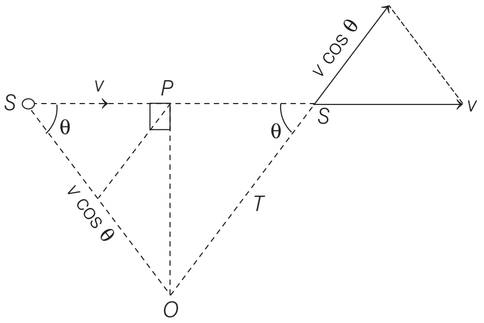
  (d) When both are moving in opposite direction, away from each other, then

  $$n' = n\left(\frac{v - v_o}{v + v_s}\right)$$



Transverse Doppler's Effect

- The Doppler's effect in sound does not take place in the transverse direction.
- As shown in figure, the position of a source is S and of observer is O. The component of velocity of source towards the observer is $v \cos\theta$. For this situation, the approach frequency is



$$\nu' = \frac{v}{v - v_s \cos\theta} \times \nu$$

ν' will now be a function of θ. So, it will no more be constant.

Similarly, if the source is moving away from the observer as shown above, with velocity component $v_s \cos\theta$, then

$$\nu' = \frac{v}{v + v_s \cos\theta} \times \nu$$

- If $\theta = 90°$, the $v_s \cos\theta = 0$ and there is no shift in the frequency. Thus, at point P, Doppler's effect does not occur.

Effect of Wind

If wind is also blowing with a velocity w in the direction of sound, then its velocity is added to the velocity of sound. Hence, in this condition the apparent frequency is given by

$$\nu' = \nu\left(\frac{v + w - v_o}{v + w - v_s}\right)$$

Applications of Doppler's Effect

The measurement of Doppler shift has been used

- by police to check over speeding of vehicles.
- at airports to guide the aircraft.
- to study heart beats and blood flow in different parts of the body.
- by astrophysicist to measure the velocities of plants and stars.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 Which of the following statements are true for wave motion?
(a) Mechanical transverse waves can propagate through all mediums
(b) Longitudinal waves can propagate through solids only
(c) Mechanical transverse waves can propagate through solids only
(d) Longitudinal waves can propagate through vacuum

2 A sound wave is passing through air column in the form of compression and rarefaction. In consecutive compression and rarefactions,
(a) density remains constant
(b) Boyle's law is obeyed
(c) bulk modulus of air oscillates
(d) there is no transfer of heat

3 A sound wave of wavelength λ is travelling in a medium with a speed of v m/s enters into another medium where its speed is $2v$ m/s. Wavelength of sound waves in the second medium is
(a) λ (b) $\frac{\lambda}{2}$
(c) 2λ (d) 4λ

4 When tension of a string is increased by 2.5 N, the initial frequency is altered in the ratio of 3 : 2. The initial tension in the string is
(a) 6 N (b) 5 N (c) 4 N (d) 2 N

5 The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal)
(a) 460 ms^{-1} (b) 500 ms^{-1} (c) 650 ms^{-1} (d) 1420 ms^{-1}

6 It takes 2.0 seconds for a sound wave to travel between two fixed points, when the day temperature is 10°C. If the temperature rise to 30°C, the sound wave travel between the same fixed points in
(a) 1.9 s (b) 2.0 s
(c) 2.1 s (d) 2.2 s

7 A wave equation is given by $y = 4\sin\left[\pi\left(\frac{t}{5} - \frac{x}{9} + \frac{1}{6}\right)\right]$
where, x is in cm and t is in sec. Which of the following is true?
(a) $\lambda = 18$ cm (b) $v = 4\ ms^{-1}$
(c) $a = 0.4$ m (d) $f = 50$ Hz

8 A wave equation which gives the displacement along y-direction is given by $y = 0.001 \sin [100t + x]$, where, x and y are in metre and t is time in second. This represents a wave

(a) of frequency $\frac{100}{\pi}$ Hz

(b) of wavelength 1 m

(c) travelling with a velocity of $\frac{50}{\pi}$ ms^{-1} in the positive x-direction

(d) travelling with a velocity of 100 ms^{-1} in the negative x-direction

9 Which of the following is not true for progressive wave

$$y = 4 \sin 2\pi \left[\frac{t}{0.02} - \frac{x}{100}\right]$$

where, y and x are in cm and t in second.

(a) Its amplitude is 4 cm

(b) Its wavelength is 100 cm

(c) Its frequency is 50 Hz

(d) Its propagation speed is 50×10^{-2} cms^{-1}

10 The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02 \text{ (m)} \sin\left[2\pi \left(\frac{t}{0.04 \text{ (s)}} - \frac{x}{0.50 \text{ (m)}}\right)\right].$$

The tension in the string is

(a) 4.0 N (b) 12.5 N (c) 0.5 N (d) 6.25 N

11 When two sound waves travel in the same direction in a medium the displacement of a particle located at X at time t is given by **→ JEE Main (Online) 2013**

$$y_1 = 0.05 \cos(0.50\pi x - 100\pi t)$$
$$y_2 = 0.05 \cos(0.46 \pi x - 92 \pi t)$$

where y_1, y_2 and x are in metres and t in seconds. The speed of sound in the medium is

(a) 92 m/s (b) 200 m/s (c) 100 m/s (d) 332 m/s

12 In order to double the frequency of the fundamental note emitted by a stretched string, the length is reduced to $\frac{3}{4}$th of the original length and the tension is changed. The factor, by which the tension is to be changed, is

(a) $\frac{3}{8}$ (b) $\frac{2}{3}$ (c) $\frac{8}{9}$ (d) $\frac{9}{4}$

13 A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations? **→ JEE Main 2018**

(a) 5 kHz (b) 2.5 kHz (c) 10 kHz (d) 7.5 kHz

14 A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water, so that half of it is in water. The fundamental frequency of the air column is now **→ JEE Main 2016 (Offline)**

(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$ (c) $2f$ (d) f

15 Third overtone of a closed organ pipe is in unison with fourth harmonic of an open organ pipe. The ratio of the lengths of the pipes are

(a) $\frac{7}{8}$ (b) $\frac{3}{4}$ (c) $\frac{5}{7}$ (d) $\frac{8}{7}$

16 Motion of two particles is given by

$$y_1 = 0.25 \sin(310t), \quad y_2 = 0.25 \sin(316t)$$

Find beat frequency.

(a) 3 (b) $3/\pi$ c) $6/\pi$ (d) 6

17 Three sound waves of equal amplitudes have frequencies $(\nu - 1), \nu, (\nu + 1)$. They superimpose to give beat. The number of beats produced per second will be

(a) 4 (b) 3 (c) 2 (d) 1

18 Two tuning forks P and Q when set vibrating, give 4 beat/s. If a prong of the fork P is filled, the beats are reduced to 2 s^{-1}. What is the frequency of P, if Q is 250 Hz?

(a) 246 Hz (b) 250 Hz (c) 254 Hz (d) 252 Hz

19 16 tuning forks are arranged in the order of increasing frequencies. Any two successive forks give 8 beat/s, when sounded together. If the frequency of the last fork is twice the first, then the frequency of the first fork is

(a) 120 Hz (b) 160 Hz
(c) 180 Hz (d) 220 Hz

20 A vehicle with a horn of frequency n is moving with a velocity of 30 ms^{-1} in a direction perpendicular to the straight line joining the observer and the vehicle. The observer perceives the sound to have a frequency $(n + n_1)$. If velocity of sound in air be 330 ms^{-1}, then

(a) $n_1 = 10n$ (b) $n_1 = 0$ (c) $n_1 = \frac{n}{11}$ (d) $n_1 = -\frac{n}{11}$

21 An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light $= 3 \times 10^8$ ms^{-1}) **→ JEE Main (Offline) 2017**

(a) 12.1 GHz (b) 17.3 GHz
(c) 15.3 GHz (d) 10.1 GHz

22 Two trains are moving towards each other at speeds of 20 ms^{-1} and 15 ms^{-1} relative to the ground. The first train sounds a whistle of frequency 600 Hz, the frequency of the whistle heard by a passenger in the second train before the train meet is
(Speed of sound in air = 340 ms^{-1})

(a) 600 Hz (b) 585 Hz (c) 645 Hz (d) 666 Hz

23 A fixed source of sound emitting a certain frequency appears as ν_a when the observer is approaching the source with speed v_o and v_r when the observer recedes from the source with the same speed. The frequency of the source is

(a) $\frac{\nu_r + \nu_a}{2}$ (b) $\frac{\nu_r - \nu_a}{2}$ (c) $\sqrt{\nu_a \cdot \nu_b}$ (d) $\frac{2\nu_r \cdot \nu_a}{\nu_r + \nu_a}$

Direction (Q. Nos. 24-31) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes* (*a*), (*b*), (*c*), (*d*) *given below:*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

24 **Statement I** A tuning fork is in resonance with a closed pipe. But the same tuning fork cannot be in resonance with an open pipe of the same length.

Statement II The same tuning fork will not be in resonance with open pipe of same length due to end correction of pipe.

25 **Statement I** In a sound wave, a displacement node is a pressure antinode and *vice-versa.*

Statement II Displacement node is a point of minimum displacement.

26 **Statement I** Velocity of particles while crossing mean position (in stationary waves) varies from maximum at antinodes to zero at nodes.

Statement II Amplitude of vibration at antinodes is maximum and at nodes, the amplitude is zero and all particles between two successive nodes cross the mean position together.

27 **Statement I** We can recognise our friends by listening their voices.

Statement II The quality of sound produced by different persons are different.

28 **Statement I** The basic of Laplace correction was that, exchange of heat between the region of compression and rarefaction in air is not possible.

Statement II Air is a bad conduction of heat and velocity of sound in air is large.

29 **Statement I** If two waves of same amplitude, produce a resultant wave of same amplitude, then the phase difference between them will be 120°.

Statement II The resultant amplitude of two waves is equal to sum of amplitude of two waves.

30 **Statement I** Two longitudinal waves given by equation $y_1(x, t) = 2a \sin(\omega t - kx)$ and $y_2(x, t) = a \sin(2\omega t - 2kx)$ will have equal intensity.

Statement II Intensity of waves of given frequency in same medium is proportional to square of amplitude only.

31. **Statement I** If we see the oscillations of a stretched wire at higher overtone mode, frequency of oscillations increases, but wavelength decreases.

Statement II From $v = \nu \cdot \lambda$, $\lambda \propto \frac{1}{\nu}$ as $v =$ constant.

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 Sound of frequency f passes through a Quinck's tube, adjusted for intensity I_m. What should be the length to which the tube should be moved to reduce intensity to 50% (speed of sound is v)?

(a) $\frac{v}{2f}$ (b) $\frac{v}{4f}$ (c) $\frac{v}{8f}$ (d) $\frac{v}{16f}$

2 A racing car moving towards a cliff sounds its horn. The driver observes that the sound reflected from the cliff has a pitch one octave higher than the actual sound of the horn. If v is the velocity of sound, the velocity of the car is

(a) $\frac{v}{\sqrt{2}}$ (b) $\frac{v}{2}$

(c) $\frac{v}{3}$ (d) $\frac{v}{4}$

3 A train of sound waves is propagated along an organ pipe and gets reflected from an open end. If the displacement amplitude of the waves (incident and reflected) are 0.002 cm, the frequency is 1000 Hz and wavelength is 40 cm. Then, the displacement amplitude of vibration at a point at distance 10 cm from the open end, inside the pipe is

(a) 0.002 cm (b) 0.003 cm (c) 0.001 cm (d) 0.000 cm

4 An engine approaches a hill with a constant speed. When it is at a distance of 0.9 km, it blows a whistle whose echo is heard by the driver after 5 s. If the speed of sound in air is 330 m/s, then the speed of the engine is

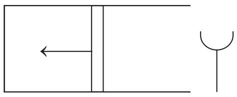
→ **JEE Main (Online) 2013**

(a) 32 m/s (b) 27.5 m/s (c) 60 m/s (d) 30 m/s

5 A travelling wave represented by $y = A\sin(\omega t - kx)$ is superimposed on another wave represented by $y = A\sin(\omega t + kx)$. The resultant is

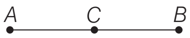
(a) a standing wave having nodes at
$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, n = 0, 1, 2$$
(b) a wave travelling along $+x$ direction
(c) a wave travelling along $-x$ direction
(d) a standing wave having nodes at $x = \frac{n\lambda}{2}$, $n = 0, 1, 2$.

6 A piston fitted in cylindrical pipe is pulled as shown in the figure. A tuning fork is sounded at open end and loudest sound is heard at open length 13 cm, 41 cm and 69 cm. The frequency of tuning fork if velocity of sound is 350 ms^{-1}, is



(a) 1250 Hz (b) 625 Hz
(c) 417 Hz (d) 715 Hz

7 A and B are two sources generating sound waves. A listener is situated at C. The frequency of the source at A is 500 Hz. A now, moves towards C with a speed 4 m/s. The number of beats heard at C is 6. When A moves away from C with speed 4 m/s, the number of beats heard at C is 18. The speed of sound is 340 m/s. The frequency of the source at B is **→ JEE Main (Online) 2013**



(a) 500 Hz (b) 506 Hz
(c) 512 Hz (d) 494 Hz

8 A pipe of length 85 cm is closed from one end.Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. **→ JEE Main 2014**

(a) 12 (b) 8 (c) 6 (d) 4

9 A sonometer wire of length 114 cm is fixed at both the ends. Where should the two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1:3:4? **→ JEE Main (Online) 2013**

(a) At 36 cm and 84 cm from one end
(b) At 24 cm and 72 cm from one end
(c) At 48 cm and 96 cm from one end
(d) At 72 cm and 96 cm from one end

10 The transverse displacement $y(x, t)$ of a wave on a string is given by $y(x, t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}\,xt)}$. This represents a

(a) wave moving in $-x$ direction with speed $\sqrt{\frac{b}{a}}$
(b) standing wave of frequency $\sqrt{b}$
(c) standing wave of frequency $\frac{1}{\sqrt{b}}$
(d) wave moving in $+x$ direction with speed $\sqrt{a/b}$

11 A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is close to (speed of sound = 320 ms^{-1}) **→ JEE Main 2015**

(a) 6% (b) 12% (c) 18% (d) 24%

12 A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (Take, $g = 10\,ms^{-2}$) **→ JEE Main (Offline) 2016**

(a) $2\pi\sqrt{2}$ s (b) 2 s (c) $2\sqrt{2}$ s (d) $\sqrt{2}$ s

13 A motor cycle starts from rest and accelerates along a straight path at 2 ms^{-2}. At the starting point of the motor cycle, there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound = 330 ms^{-1})

(a) 49 m (b) 98 m (c) 147 m (d) 196 m

ANSWERS

SESSION 1	**1** (c)	**2** (d)	**3** (c)	**4** (d)	**5** (d)	**6** (a)	**7** (a)	**8** (d)	**9** (d)	**10** (d)
	11 (b)	**12** (d)	**13** (a)	**14** (d)	**15** (a)	**16** (b)	**17** (c)	**18** (a)	**19** (a)	**20** (b)
	21 (b)	**22** (d)	**23** (a)	**24** (c)	**25** (b)	**26** (a)	**27** (a)	**28** (a)	**29** (c)	**30** (c)
	31 (a)									
SESSION 2	**1** (c)	**2** (c)	**3** (d)	**4** (d)	**5** (a)	**6** (b)	**7** (c)	**8** (c)	**9** (b)	**10** (a)
	11 (b)	**12** (c)	**13** (b)							

Hints and Explanations

SESSION 1

1 Mechanical transverse wave can be set up in solids but never in liquids and gases.

2 There is no transfer of heat from compression to rarefaction as air is a bad conductor of heat and time of compression and rarefaction is too small.

3 In the first medium, frequency

$$f = \frac{c}{\lambda} = \frac{v}{\lambda}$$

It remains the same in second medium,

i.e. $f' = f$

$$\frac{v'}{\lambda'} = \frac{2v}{\lambda'} = \frac{v}{\lambda} \Rightarrow \lambda' = 2\lambda$$

4 Since, frequency, $\nu \propto \sqrt{T}$

$\therefore \quad \frac{\nu_1}{\nu_2} = \sqrt{\frac{T_1}{T_2}}$ or $\frac{3}{2} = \sqrt{\frac{T + 2.5}{T}}$

or $\frac{9}{4} = 1 + \frac{2.5}{T}$ or $\frac{2.5}{T} = \frac{5}{4}$ or $T = 2\,\text{N}$

5 Speed of sound is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$v_{O_2} = \sqrt{\frac{\frac{7}{5}RT}{32}} \text{ and } v_{He} = \sqrt{\frac{\frac{5}{3}RT}{4}}$$

$$\therefore \frac{v_{O_2}}{v_{He}} = \sqrt{\frac{7 \times 3 \times 4}{5 \times 32 \times 5}}$$

or $v_{He} = 460 \times \sqrt{\frac{5 \times 32 \times 5}{7 \times 3 \times 4}} \approx 1420\ \text{ms}^{-1}$

6 Let distance between two fixed points be d, then

$$t = \frac{d}{v} \text{ also } v \propto \sqrt{T}$$

$$\Rightarrow \quad \frac{t_1}{t_2} = \frac{v_1}{v_2} = \sqrt{\frac{T_2}{T_1}}$$

$$\Rightarrow \quad \frac{2}{t_2} = \sqrt{\frac{303}{283}} \Rightarrow t_2 = 1.9\ \text{s}$$

7 The given equation be written as

$$y = 4 \sin\left[\pi\left(\frac{t}{5} - \frac{x}{9} + \frac{1}{6}\right)\right] \quad \ldots\text{(i)}$$

The standard wave equation can be written as

$$y = a \sin(\omega t - kx + \phi)$$

$$y = a \sin\left(\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x + \phi\right) \ldots\text{(ii)}$$

Equating Eqs. (i) and (ii), we get

Amplitude, $\quad a = 4\ \text{cm}$

Frequency, $\quad f = \frac{1}{T} = \frac{1}{10}\ \text{Hz} = 0.1\ \text{Hz}$

Wavelength, $\lambda = 2 \times 9 = 18$ cm

Velocity, $\quad v = f\lambda$

$\quad = 0.1 \times 18 = 1.8\ \text{cms}^{-1}$

8 Comparing given equation with standard form of wave equation, we get

$$A = 0.001\ \text{m}, \omega = 100\ \text{s}^{-1}$$

and $k = 1\ \text{m}^{-1}$

$$\therefore \quad \nu = \frac{100}{2\pi} = \frac{50}{\pi}\ \text{Hz}$$

$$\lambda = \frac{2\pi}{k} = 2\pi\ \text{m}$$

and $v = \frac{\omega}{k} = \frac{100\ \text{s}^{-1}}{1\ \text{m}^{-1}} = 100\ \text{ms}^{-1}$

Moreover, as the wave equations of the form $y = A\sin(\omega t + kx)$, the wave is travelling along negative x-direction.

9 From the given wave equation, we find that

$$A = 4\ \text{cm}, \omega = \frac{2\pi}{0.02}\ \text{s}^{-1} = 100\,\pi\text{s}^{-1}$$

and $k = \frac{2\pi}{100}\ \text{cm}^{-1}$

$$\therefore \quad \lambda = \frac{2\pi}{k} = 100\ \text{cm}$$

$$\nu = \frac{\omega}{2\pi} = \frac{100\pi}{2\pi} = 50\ \text{Hz}$$

and $v = \frac{\omega}{k} = \frac{100\pi}{\frac{2\pi}{100}} = 50 \times 10^2\ \text{cms}^{-1}$

Hence, answer of wave speed v is wrong, i.e. option (d) is correct.

10 Tension in the string is given by

$$T = \mu v^2 = \mu \frac{\omega^2}{k^2}$$

$$= \frac{0.04\,(2\pi / 0.04)^2}{(2\pi / 0.50)^2} = 6.25\text{N}$$

11 $y_1 = 0.05\cos(0.50\,\pi\,x - 100\pi\,t)$

and $y_2 = 0.05\cos(0.46\,\pi\,x - 92\,\pi t)$

Comparing these two equations with $y = A\sin(k\,x - \omega t)$

We have, $\omega_1 = 100\,\pi$ and $\omega_2 = 92\,\pi$

Now, speeds, $v_1 = \frac{A}{100\pi} = \frac{0.05}{100\,\pi}$

and $\quad v_2 = \frac{A}{92\,\pi} = \frac{0.05}{92\,\pi}$

Now, the resultant speed,

$$v = \sqrt{\left(\frac{0.05}{92\,\pi}\right)^2 + \left(\frac{0.05}{100\,\pi}\right)^2} \quad [\pi = 3.14]$$

$$= 200\ \text{m/s}$$

12 From law of string, other factors remaining unchanged,

$$\frac{\nu_1}{\nu_2} = \frac{l_2}{l_1}\sqrt{\frac{T_1}{T_2}}$$

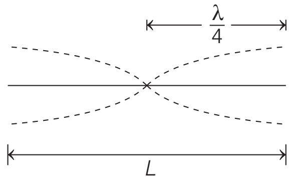
Now, $\nu_2 = 2\nu_1, l_2 = \frac{3}{4} l_1$,

Hence, we have

$$\frac{1}{2} = \frac{3}{4} \times \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{2}{3} = \sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{9}{4} \text{ or } T_2 = \frac{9}{4} T_1$$

13



From vibration mode,

$$\frac{\lambda}{2} = L$$

$$\Rightarrow \quad \lambda = 2L$$

$\therefore$ Wave speed, $v = \sqrt{\frac{Y}{\rho}}$

So, frequency

$$f = \frac{v}{\lambda}$$

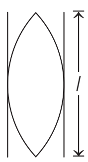
$$\Rightarrow \quad f = \frac{1}{2L}\sqrt{\frac{Y}{\rho}}$$

$$= \frac{1}{2 \times 60 \times 10^{-2}}\sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}}$$

$$\approx 5000\ \text{Hz}$$

$$f = 5\ \text{kHz}$$

14 For open ends, fundamental frequency f in air, we have



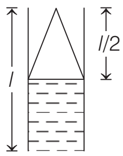
$$\frac{\lambda}{2} = l$$

$$\Rightarrow \quad \lambda = 2l$$

$$v = f\lambda$$

$$\Rightarrow \quad f = \frac{v}{\lambda} = \frac{v}{2l} \quad \ldots\text{(i)}$$

When a pipe is dipped vertically in water, so that half of it is in water, we have



$$\frac{\lambda}{4} = \frac{l}{2}$$

$$\Rightarrow \quad \lambda = 2l \Rightarrow v = f'\lambda$$

$$\Rightarrow \quad f' = \frac{v}{\lambda} = \frac{v}{2l} = f \quad \ldots\text{(ii)}$$

Thus, the fundamental frequency of the air column is now,

$$f = f'$$

15 We know that, third overtone of closed organ pipe means seventh harmonic

$\therefore \quad (\nu_7)_{closed} = (\nu_4)_{open}$

or $\quad 7\left(\dfrac{v}{4l_c}\right) = 4\left(\dfrac{v}{2l_o}\right)$

or $\quad \dfrac{l_o}{l_c} = \dfrac{8}{7}$

or $\quad \dfrac{l_c}{l_o} = \dfrac{7}{8}$

16 $y_1 = 0.25 \sin(310t)$ ...(i)

and $y_2 = 0.25 \sin(316t)$ ...(ii)

We have, $\omega_1 = 310$

$\Rightarrow \quad f_1 = \dfrac{310}{2\pi}$ unit

and $\quad \omega_2 = 316$

$\Rightarrow \quad f_2 = \dfrac{316}{2\pi}$ unit

Hence, beat frequency

$= f_2 - f_1 = \dfrac{316}{2\pi} - \dfrac{310}{2\pi}$

$= \dfrac{3}{\pi}$ unit

17 Maximum number of beats $= \nu + 1 - (\nu - 1) = 2$

18 There are four beats between P and Q, therefore the possible frequencies of P are 246 or 254 (i.e. 250 ± 4) Hz.

When the prong of P is filled, its frequency becomes greater than the original frequency.

If we assume that the original frequency of P is 254, then on filling its frequency will be greater than 254. The beats between P and Q will be more than 4. But it is given that the beats are reduced to 2, therefore, 254 is not possible. Therefore, the required frequency must be 246 Hz.

19 As forks have been arranged in ascending order of frequencies, hence if frequency of Ist fork be n, then

$n_2 = n + 8$

$n_3 = n + 2 \times 8 = n + 16$

and $\quad n_{16} = n + 15 \times 8$

$= n + 120 = 2n$

$\Rightarrow \quad n = 120$ Hz

20 Since, vehicle having siren is moving in a perpendicular direction, hence there will be no Doppler shift in frequency and $n_1 = 0$.

21 As the observer is moving towards the source, so frequency of waves emitted by the source will be given by the formula,

$$f_{observed} = f_{actual} \cdot \left(\frac{1 + v/c}{1 - v/c}\right)^{1/2}$$

Here, frequency, $\dfrac{v}{c} = \dfrac{1}{2}$

So, $f_{observed} = f_{actual}\left(\dfrac{3/2}{1/2}\right)^{1/2}$

$\therefore \quad f_{observed} = 10 \times \sqrt{3} = 17.3$ GHz

22 Here, $v = 340 \text{ ms}^{-1}$,

$v_s =$ velocity of 1st train $= 20 \text{ ms}^{-1}$,

$v_o =$ velocity of 2nd train $= 15 \text{ ms}^{-1}$ and

$\nu_0 = 600$ Hz.

As S and O are approaching each other, hence

$$\nu = \left[\frac{v + v_o}{v - v_s}\right]\nu_0$$

$$= \left(\frac{340 + 15}{340 - 20}\right) \times 600 = 666 \text{ Hz}$$

23 Here, $\nu_a = \nu\left(\dfrac{v + v_o}{v}\right)$

or $\quad \dfrac{\nu_a}{\nu} = 1 + \dfrac{v_o}{v}$

or $\quad \dfrac{v_o}{v} = \dfrac{\nu_a}{\nu} - 1$ ...(i)

Again, $\nu_r = \nu\left(\dfrac{v - v_o}{v}\right)$

$\therefore \quad \dfrac{v_o}{v} = 1 - \dfrac{\nu_r}{\nu}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{\nu_a}{\nu} - 1 = 1 - \frac{\nu_r}{\nu}$$

or $\quad \dfrac{\nu_a}{\nu} + \dfrac{\nu_r}{\nu} = 2$

or $\quad \dfrac{\nu_a + \nu_r}{\nu} = 2$

or $\quad \nu = \dfrac{\nu_a + \nu_r}{2}$

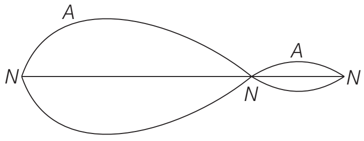
24 If a closed pipe of length L is in resonance with a tuning fork of frequency ν, then $\nu = \dfrac{v}{4L}$

An open pipe of same length L produces vibrations of frequency $\dfrac{v}{2L}$. Obviously, it cannot be in resonance with the given tuning fork of frequency $\nu\left(=\dfrac{v}{4L}\right)$.

25 At the point where a compression and a rarefaction meet, the displacement is minimum and it is called displacement node. At this point, the pressure difference is maximum i.e. at the same time, it is a pressure antinode.

On the other hand, at the mid-point of a compression or a rarefaction, the displacement variation is maximum i.e. such a point is displacement antinode. However such a point is pressure node, as pressure variation is minimum at such a point.

26 Stationary wave is represented as shown in figure.



It is quite clear from figure that at nodes the amplitude is zero and velocity of particle is also zero and at antinodes the amplitude is maximum. So that the velocity of particle is also maximum and all particles cross mean position between two successive nodes.

27 Sounds coming from the different sources can be recognised by virtue of their quality which is characteristics of sound. That is why we recognise the voices of our friends.

28 According to Laplace, the changes in pressure and volume of a gas, when sound waves propagated through it, are not isothermal but adiabatic. A gas is a bad conductor of heat. It does not allow the free exchange of heat between compressed layer, rarified layer and its surrounding.

29 The resultant amplitude of two waves is given by

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\theta}$$

Here, $a_1 = a_2 = A = a$

$1/2 = 1 + \cos\theta$

or $\cos\theta = -1/2$ or $\theta = 120°$

30 $I = \dfrac{1}{2}\rho\omega^2 A^2 v$

Here, $\rho =$ density of medium,

$A =$ amplitude,

$\omega =$ angular frequency and

$v =$ velocity of wave

$\therefore$ Intensity depend upon amplitude, frequency as well as velocity of wave

Also, $\quad I_1 = I_2$

31 $\nu = \eta\left(\dfrac{v}{2l}\right)$, where $n = 1, 2, 3, \ldots$

As, η increases, frequency increases. Hence, wavelength decreases.

SESSION 2

1 $I_m = 4ka^2, I = 2ka$

$I' = k(a^2 + a^2 + 2a^2 \cos^2 \theta)$

$\Rightarrow \quad \phi = \frac{\pi}{2}$

Path difference $= \frac{\lambda}{4} = 2x$

$\Rightarrow \quad \frac{v}{4f} = 2x$

$\Rightarrow \quad x = \frac{v}{8f}$

2 Let n be the actual frequency of sound of horn. If v_s be the velocity of car, then frequency of sound striking the cliff (source is moving towards listener)

$$n' = \frac{v \times n}{v - v_s} \quad \ldots(i)$$

The frequency of sound heard on reflection

$$n'' = \frac{(v + v_s)n'}{v} = \frac{(v + v_s)}{v} \times \frac{v \times n}{(v - v_s)}$$

or $\quad \frac{n''}{n} = \frac{v + v_s}{v - v_s} = 2$

$v + v_s = 2v - 2v_s$

or $\quad v_s = \frac{v}{3}$

3 The equation of stationary wave for open organ pipe can be written as,

$y = 2A \cos\left(\frac{2\pi x}{\lambda}\right) \sin\left(\frac{2\pi ft}{v}\right)$,

where x is the open end from where wave gets reflected.

Amplitude of stationary wave is,

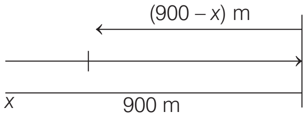
$A_s = 2A \cos\left(\frac{2\pi x}{\lambda}\right)$

For $x = 0.1$ m,

$A_s = 2 \times 0.002 \cos\left[\frac{2\pi \times 0.1}{0.4}\right] = 0$

4 We have, $900 + 900 - x$

$= 330 \times 5 = 1650$



$\therefore \quad x = 150$ m

$\therefore$ Speed $= \frac{150}{5} = 30$ m/s

5 $y = y_1 + y_2$

$= A \sin(\omega t - kx) + A \sin(\omega t + kx)$

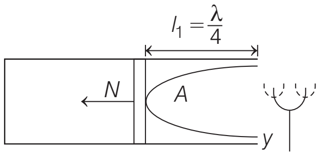
$y = 2A \sin \omega t \cos kx$

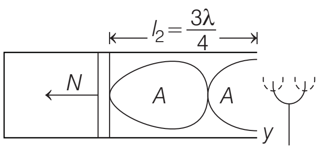
Clearly, it is equation of standing wave for position of nodes $y = 0$.

i.e. $\quad x = (2n + 1)\frac{\lambda}{4}$

$= \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$

6 In a closed organ pipe in which length of air-column can be increased or decreased, the first resonance occurs at $\lambda/4$ and second resonance occurs at $3\lambda/4$.





Thus, at first resonance,

$$\frac{\lambda}{4} = 13 \quad \ldots(i)$$

and at second resonance,

$$\frac{3\lambda}{4} = 41 \quad \ldots(ii)$$

Subtracting Eq. (i) from Eq. (ii), we have

$$\frac{3\lambda}{4} - \frac{\lambda}{4} = 41 - 13$$

$\therefore \quad \lambda = 56$ cm

Hence, frequency of tuning fork,

$$n = \frac{v}{\lambda} = \frac{350}{56 \times 10^{-2}} = 625\text{Hz}$$

7 Here, frequency of source $= 500$ Hz

Speed of source $A = 4$ m/s $= u$

Then, source is moving towards stationary observer,

$v' = \frac{v}{v - u} v_0$

(where, v = speed of sound)

$= \frac{340}{340 - 4} \times 500$

$\Rightarrow \quad v' = \frac{340}{336} \times 500$ Hz

$= 506$ Hz

Now, when source is reciding from the observer

$v' = \frac{v}{v + u} v_0$

$= \frac{340}{344} \times 500$ Hz

$\therefore \quad v' = 494$ Hz

According to question,

Let freqency of source B is Z Hz

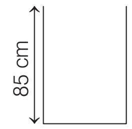
$\therefore \quad Z = 506 \pm 6 \Rightarrow Z = 500$ or 512

and $Z = 494 \pm 18 \Rightarrow Z = 512$ or 476

Thus, required frequency $= 512$ Hz

8 For closed organ pipe $= \frac{(2n + 1)v}{4l}$

[where, $n = 0,1,2.....$]



$\frac{(2n + 1)v}{4l} < 1250$

$(2n + 1) < 1250 \times \frac{4 \times 0.85}{340}$

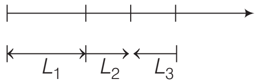
$(2n + 1) < 12.52$

$n < 5.25$

So, $n = 0,1,2,3,\ldots,5$

So, we have 6 possibilities.

9 $v_1 = \frac{1}{2L_1}\sqrt{\frac{f}{\mu}}$



Let length of three segments be L_1, L_2 and L_3,

$v_2 = \frac{1}{2L_2}\sqrt{f/\mu}$

$v_3 = \frac{1}{2L_3}\sqrt{f/\mu}$

So, that $\quad v_1 L_1 = v_2 L_2 = v_3 L_3$

As, $\quad v_1 : v_2 : v_3 = 1 : 3 : 4$

$v_1 = \frac{v_2 L_2}{L_1}$

and $\quad v_2 = 3v_1, v_3 = 4v_1$

$L_2 = \frac{v_1}{v_2} L_1 = \frac{1}{3} L_1 = \frac{L_1}{3}$

and $L_3 = \frac{v_1}{v_3} L_1 = \frac{v_1}{4v_1} L_1 = \frac{L_1}{4}$

$L_1 + L_2 + L_3 = 114$

Now, $L_1\left(1 + \frac{1}{3} + \frac{1}{4}\right) = 114$

$\Rightarrow \quad L_1\left(\frac{12 + 4 + 3}{12}\right) = 114$

$L_1\left(\frac{19}{12}\right) = 114$

$\Rightarrow \quad L_1 = \frac{(114 \times 12)}{19} = 72$ cm

$L_2 = \frac{L_1}{3} = \frac{72}{3} = 24$ cm

10 $y(x,t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}xt)}$

$= e^{-(\sqrt{a}x + \sqrt{b}t)^2}$

It is a function of type

$y = f(\omega t + kx)$

$\therefore$ $y(x,t)$ represents wave travelling along $-x$ direction.

Speed of wave $= \dfrac{\omega}{k} = \dfrac{\sqrt{b}}{\sqrt{a}} = \sqrt{\dfrac{b}{a}}$

11 Apparent frequency heard by the person before crossing the train.

$f_1 = \left(\dfrac{c}{c - v_s}\right) f_o = \left(\dfrac{320}{320 - 20}\right) 1000$

Similarly, apparent frequency heard,after crossing the trains

$f_2 = \left(\dfrac{c}{c + v_s}\right) f_o = \left(\dfrac{320}{320 + 20}\right) 1000$

[c = speed of sound in air]

$\Delta f = f_1 - f_2$

$= \left(\dfrac{2cv_s}{c^2 - v_s^2}\right) f_0$

Percentage change in frequency heard by the person standing near the track as the train passes him is

or $\dfrac{\Delta f}{f_0} \times 100 = \left(\dfrac{2cv_s}{c^2 - v_s^2}\right) \times 100$

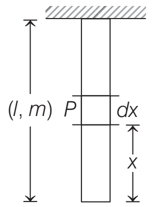
$= \dfrac{2 \times 320 \times 20}{300 \times 340} \times 100$

$= \dfrac{2 \times 32 \times 20}{3 \times 34}$

$= 12.55\% = 12\%$

12 A uniform string of length 20 m is suspended from a rigid support. Such that the time taken to reach the support,

$T = \dfrac{mgx}{l}$



So, velocity at point $P = \sqrt{\dfrac{\frac{mgx}{l}}{m/l}}$

i.e. $v = \sqrt{gx}$

$\dfrac{dx}{dt} = \sqrt{gx}$

$\Rightarrow$ $\displaystyle\int_0^{20} \frac{dx}{\sqrt{x}} = \int_0^t \sqrt{g}\, dt$

$[2\sqrt{x}]_0^{20} = \sqrt{10}t$

$\Rightarrow$ $2\sqrt{20} = \sqrt{10}t$

$t = 2\sqrt{2}$ s

13 For motor cycle, $u = 0, a = 2$ ms^{-2}

Observer is in motion and source is at rest, then apparent frequency,

$\nu' = \nu \dfrac{v - v_o}{v + v_s}$

$\Rightarrow$ $\dfrac{94}{100} \nu = \nu \dfrac{330 - v_o}{330}$

$\Rightarrow$ $330 - v_o = \dfrac{330 \times 94}{100}$

$\Rightarrow$ $v_o = 330 - \dfrac{94 \times 33}{10}$

$= \dfrac{33 \times 6}{10}$ ms^{-1}

$s = \dfrac{v^2 - u^2}{2a}$

$= \dfrac{9 \times 33 \times 33}{100}$

$= \dfrac{9 \times 1089}{100} \approx 98$ m